# INFLUENCE OF RADIATION DAMPING ON THE SCATTERING OF PSEUDOSCALAR CHARGED MESONS BY NUCLEONS

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**ABSTRACT.** In the present paper it is shown that the inclusion of the influence of radiation damping on the scattering of $\pi^+$ meson by proton explains satisfactorily the variation of the total scattering cross section with energy of the incident $\pi^+$ meson as experimentally observed by Anderson and by Sachs and Steinberger; for a proper fit to the experimental curve the value of the coupling constant $g^2$ is 0.56. The well-known Heitler's integral equation for the above problem is, however, solved by the semi-variational method of Ma and Hsueh, because it is not possible to find an exact solution in this case. The corresponding integral equation for the scattering of $\pi^-$ meson by proton has been exactly solved by Ma; a comparison of Ma's result with ours shows that experimentally observed ratio of the cross section of scattering by proton of $\pi^+$ meson to that of $\pi^-$ meson is explained by the weak coupling perturbation method which includes radiation reactions.

## 1. INTRODUCTION

It has been mentioned by Bhabha (1940) and Heitler (1941) that the theory of radiation damping plays a vital role in the meson theory. In fact, the wellknown discrepancy between the theoretical and the experimental results for the cross section of scattering of mesons by nucleons can be removed if we consider the effect of radiation damping. Attempts have been made by several authors (Heitler, Wilson, Peng and Gora) to take into account this field reaction. They have replaced the transition matrix element $H_{fi}$ (in the Born approximation) by $U_{fi}$, where $U_{fi}$ is the solution of the following integral equation:

$$U_{fi} = H_{fi} - i\pi \sum_{f'} \int H_{ff'} \rho_{f'} U_{f'i} \, d\Omega_{f'}$$

Exact solutions of this integral equation have been obtained by Heitler (1941) in the non-relativistic approximation. Since the influence of radiation damping is important only at high energies, it is necessary to have an exact relativistic treatment of the problem. The solution of the integral equation in such cases is mathematically very complicated. Exact solutions of this integral equation can only be had in some particular cases when the kernel of this non-homogenous equation is degenerate. The formal method of solution of the integral equation with degenerate kernel is to transform the equation to a system of algebraic equations by a suitable transformation. By this method, Ma (1943) and Hsueh and Ma (1944) have solved the integral equation for the scattering of a positive meson by neutron and that of a negative meson by proton.

Here we shall consider the scattering of a positive meson by a proton and that of a negative meson by neutron. In this case the kernel $H_{ff'}$ of the integral equation is non-degenerate. So the method of solving the integral equation by transforming it into a system of algebraic equations is not applicable here. Other methods, such as Fourier and Mellin transformations, are not applicable owing to the serious complicacy of the kernel.

The general series solution (Fredholm's series or Louville-Newman's method of iteration) does not always yield tenable results. The difficulty lies in the fact that the resulting iterated series cannot in all cases be summed and even in most relativistic cases, the calculations of higher order terms of the series are exceedingly involved due to spur calculations. It is seen that this particular mothod is only applicable in the case of non-relativistic treatment of scattering of light by electron (Thomson's formula). In this case the result agrees with the exact solution obtained by Heitler. In the relativistic scattering of the pseudoscalar charged positive meson by proton, this method is absolutely untenable.

In view of this difficulty, it is desirable to find approximate solutions of the problem. Of the approximate solutions proposed by Wilson and Hsueh and Ma, the semi-variational procedure of the latter is more reliable. By this approximate solution Basu (1949, 1951) in the scattering of neutron by proton at high energies has obtained a result which is in good agreement with the available experimental results. So we apply here this semi-variational procedure in the scattering of positive meson by proton and that of negative meson by neutron.

Recently, using pseudoscalar meson field, Corinaldesi and Field (1949, (1950) attempted to take into consideration this field reaction in the non-relativistic scattering of positive meson by proton. But they have not solved the integral equation rigourously. Only for qualitative analysis they have replaced the transition matrix element $U_{fi}$ in the damping term by an average over all angles. But this is not mathematically justifiable.

In this paper we shall assume the pseudoscalar meson field and our calculation has been performed in the case of pseudoscalar $g_2$ coupling only. In the last section a comparison of the theoretical results with the experimental one obtained by Anderson and Steinberger (1951,1951) has been made by means of a graph. The graph shows the energy dependence of the damped cross section. A higher value of $g_2$ has been suggested in order that the theoretically obtained damped crosss ection to be in good agreement with the experimental one. Another feature of the curves drawn may be mentioned. It has been shown that the damped cross section of the positive meson by proton is larger than that of the negative meson by proton. This is also an experimental fact.

For simplification, we have used throughout $\hbar = c = 1$.

METHOD AND SOLUTION

The matrix element $U_{fi}$ which determines the transition from an initial state $i$ to a final state $f$, in the theory of radiation damping is the solution of the following integral equation :

$$U_{fi} = H_{fi} - i\pi \sum_{f'} \int H_{ff'} U_{f'i} \rho_{f'} d\Omega_{f'} \qquad \dots \quad (1)$$

Here $\rho_{f'}$ denotes the density of the energy level corresponding to the state $f'$. $H_{fi}$ is the scattering amplitude in the ordinary perturbation theory. The integration and summation are to be carried out over all directions and polarisations of the state $f'$.

As mentioned above, in this particular case of the scattering of a positive meson by a proton, the kernel $H_{ff'}$ of the equation (1) is non-degenerate. So exact solutions cannot be had. We therefore look for an approximate solution of (1) by the semi-variational method proposed by Hseuh and Ma (1945).

An equivalent form of equation (1) is the following :

$$\sum_{i} \sum_{f} \int \delta U^*_{if} \left[ U_{fi} - H_{fi} + \pi i \sum_{f'} \int H_{ff'} U_{f'i} \rho_{f'} d\Omega_{f'} \right] \rho_f d\Omega_f = 0 \qquad \dots \quad (2)$$

where $\delta U^*_{if}$ is the arbitrary variation of the complex conjugate of $U_{fi}$. Equation (1) holds for all cases if (2) is satisfied for any arbitrary variation of $\delta U^*_{if}$.

Let us assume a trial solution $xH_{fi}$ of $U_{fi}$

*i.e.* $$U_{fi} = xH_{fi} \qquad \dots \quad (3)$$

where $x$ is a parameter not depending on the final or initial state, $f$ or $i$. Then equation (2) must hold good for the solution $xH_{fi}$ of $U_{fi}$.

Substituting (3) in (2) and varying $x$ we get

$$\delta x^* \sum_{i} \sum_{f} \int H_{fi} \left[ H_{if}(x-1) + i\pi \sum_{f'} \int H_{ff'} H_{f'i} \rho_{f'} \, d\Omega_{f'} \right] \rho_f \, d\Omega_f = 0 \qquad \dots \quad (4)$$

On solving for $x$ we have

$$x = \frac{a}{a + ib} \qquad \dots \quad (5)$$

where

$$a = \sum_{i} \sum_{f} \int H_{if} H_{fi} \rho_f d\Omega_f \qquad (6a)$$

and $$b = \pi \sum_i \sum_f \sum_{f'} \iint H_{if} H_{ff'} H_{f'i} \rho_f \rho_{f'} d\Omega_f d\Omega_{f'} \tag{6b}$$

where again each of these processes, $H_{fi}$, $H_{ff'}$, etc takes place in two successive stages through some intermediate states which we denote by $n$, $n'$ and $n''$. Hence from (6) we write

$$a = \sum_i \sum_f \sum_n \sum_{n'} \int \frac{H_{in} H_{nf} H_{fn'} H_{n'i}}{(E_i - E_n)(E_i - E_{n'})} \rho_f d\Omega_f \tag{7a}$$

and

$$b = \pi \sum_i \sum_f \sum_{f'} \sum_n \sum_{n'} \sum_{n''} \iint \frac{H_{in} H_{nf} H_{fn'} H_{n'f'} H_{f'n''} H_{n''i}}{(E_i - E_n)(E_i - E_{n'})(E_i - E_{n''})} \rho_f \rho_{f'} d\Omega_{f'} d\Omega_f \quad \ldots \quad (7b)$$

Now for the pseudoscalar charged meson field, the total Hamiltonian density is

$$H_1 = \pi^* \pi + (\text{grad}\ \psi^* .\ \text{grad}\ \psi) + \mu^2 \psi^* \psi \quad \ldots \quad (8)$$

and for the nucleon field,

$$H_2 = \phi_i^* \left[ \frac{1}{i} \alpha . \text{grad} + \beta . M \right] \phi \quad \ldots \quad (9)$$

where $\alpha$ and $\beta$ are the Dirac-matrices and the mass of the nucleon is $M$. $\phi$ is an eight-component column matrix :

$$\phi = \begin{pmatrix} \phi_P \\ \phi_N \end{pmatrix} \text{ where } \phi_P = \begin{pmatrix} \phi_1 \\ \vdots \\ \phi_4 \end{pmatrix} \text{and } \phi_N = \begin{pmatrix} \phi_5 \\ \vdots \\ \phi_8 \end{pmatrix} \quad \ldots \quad (10)$$

The Hamiltonian density for the interaction field (in case of pseudoscalar $g_2$-coupling only) is given by

$$H_i = -\sqrt{(4\pi)} g \{ (\phi^\dagger \gamma_5 \tau_{PN} \phi) \psi + (\psi^\dagger \gamma_5 \tau_{NP} \phi^*) \psi^* \} \quad \ldots \quad (11)$$

where
$$\left. \begin{aligned} \phi^\dagger &= i \phi^* \beta \\ \gamma_5 &= \gamma_1 \gamma_2 \gamma_3 \gamma_4 \\ \tau_{PN} &= \begin{pmatrix} 01 \\ 00 \end{pmatrix} \text{and } \tau_{NP} = \begin{pmatrix} 00 \\ 10 \end{pmatrix} \end{aligned} \right\} \quad \ldots \quad (12)$$

Here we will discuss the process involving positive meson by proton. The second order process is schematically given by

$$\left. \begin{aligned} & Y^+(\mathbf{p}) + P(-\mathbf{p}) + [N(-\mathbf{p} - \mathbf{p}')] \\ & \longrightarrow P(-\mathbf{p}') + P(-\mathbf{p}) \\ & \longrightarrow P(-\mathbf{p}') + y^+(\mathbf{p}') + [N(-\mathbf{p} - \mathbf{p}')] \end{aligned} \right\} \quad \ldots \quad (13)$$

where the frame of reference has been so chosen that the momenta of two particles are equal and opposite.

In the above scheme $Y^+(\mathbf{p})$ represents a positive meson particle with momentum $p$; $P(-p)$ represents a proton with momentum $-\mathbf{p}$ and $N(-p-p')$

in the square bracket stands for neutron with momentum $(-\mathbf{p}-\mathbf{p}')$ in the negative energy state.

The second-order matrix element for the above process (13) is given by the theory of perturbation (Corinaldesi and Field).

$$H_{fi}=a^*_f(\mathbf{P}_f)H'a_i(\mathbf{P}_i) \qquad \ldots \quad (14)$$

where $a(p)$ satisfies the Dirac equation for the nucleon

$$[(\alpha.\mathbf{P})+\beta M]a(P)=Ea(\mathbf{P})$$

and

$$H'=\frac{2\pi g^2}{V\sqrt{\epsilon_i\epsilon_f}}\left\{\frac{\epsilon_f-E_i+\alpha\mathbf{P}+M\beta}{(E_i-\epsilon_f)^2-E^2}\right\} \qquad \ldots \quad (15)$$

where $V$ is the periodicity volume, $\mathbf{P}=\mathbf{p}_i-\mathbf{p}_f$, $\epsilon_i=\sqrt{(p_i{}^2+\mu^2)}$, $\epsilon_f=\sqrt{(p_f{}^2+\mu^2)}$, $E_i=\sqrt{(P_i{}^2+M^2)}$; $E_f=\sqrt{(P_f{}^2+M^2)}$, $E=\sqrt{(P^2+M^2)}$ and $\mathbf{P}$ stands for the nucleon momentum and $\mathbf{p}$ for the meson momentum.

In order to simplify calculations we take the reference system in which the centre of gravity of the proton and meson is at rest. So

$$\mathbf{P}_i=-\mathbf{p}_i\ ;\ \mathbf{P}_f=-\mathbf{p}_f$$

We shall also use the conservation of energy between the initial and the final states. Hence it follows that

$$|\mathbf{P}_i|=|\mathbf{p}_i|=|\mathbf{P}_f|=|\mathbf{p}_f|=p$$

and

$$\epsilon_i=\epsilon_f=\epsilon\ ;\ E_i=E_f=E$$

and the angle between the initial and the final directions of meson is $\theta$ *i.e.*

$$(\mathbf{p}_i\,.\,\mathbf{p}_f)=p^2\cos\theta$$

With these above simplifications, the matrix element for the process under investigation reduces to

$$H'_{fi}=-\frac{2\pi g^2}{V.\epsilon}a^*_f\left\{\frac{(\epsilon-E)-\alpha(\mathbf{p}_i+\mathbf{p}_f)+M\beta}{(E-\epsilon)^2-E^2}\right\}a_i$$

The evaluation of the expressions $a$ and $b$ given in ($6a$, $6b$) with (16) as the matrix element is laborious but straightforward. We shall use the usual method of summing over the spin states. Thus

$$a=\frac{(2\pi g^2)^2}{2V^2\epsilon^2E^2}\rho_f\int\{(A+B\cos\theta_{if})/[X-2p^2\cos\theta_{if}]^2\}\,d\Omega_f \qquad \ldots \quad (17)$$

where

$$A=\mu^4\left[\frac{E^2}{\mu^2}\cdot\frac{(\epsilon-E)^2}{\mu^2}+\frac{M^2}{\mu^2}\cdot\frac{(\epsilon-E)^2}{\mu^2}+2x^4+\frac{M^4}{\mu^4}\right.$$
$$\left.+2x^2\frac{E^2}{\mu^2}-4\frac{E^3}{\mu^3}\frac{(E-\epsilon)}{\mu}\right]$$

$$B=\mu^4\left[x^2(3x^2+1)+x^2.\frac{E.\epsilon}{\mu^2}\right.$$

$$X=(E-\epsilon)^2-M^2-2p^2$$

and $x$ stands for the ratio $p/\mu$, $p$ is the meson momentum and $\mu$ is the meson mass

and

$$b=-\left(\frac{2\pi g^2}{V\epsilon}\right)\rho_i\rho_f\pi\iint\frac{(F_1+F_2+F_3+F_4)\,d\Omega_f\,d\Omega_{f'}}{4E^3(X-2p^2\cos\theta_{if})(X-2p^2\cos\theta_{if'})(X-2p^2\cos\theta_{ff'})} \qquad \ldots \quad (18)$$

where

$$\begin{aligned}
F_1=\big[&3E^2(\epsilon-E)^3+(\epsilon-E)^2\{2M^2E^2+M^2(2E\epsilon-p^2)+2E^2p^2\}\\
&+(2\epsilon-E)\{E^2(\epsilon-E)(E^2+M^2)+2M^2E^3\}\\
&+(\epsilon-E)\{2M^2E(2E\epsilon-p^2)+2Mp^2E^2+3p^4E\}\\
&+E^2\epsilon^2p^2+M^2E^2(2E\epsilon-p^2)+3E\epsilon p^4+2M^2p^2\epsilon^2\\
&+p^2(E\epsilon-p^2)^2+M^2p^2E\epsilon+p^4E^2+p^6\big]\\
F_2=p^2\big[&(E\epsilon-E)^3+(\epsilon-E)^2(4E^2-M^2)+(\epsilon-E)\{2E^3+Ep^2\\
&+M^2E+(2E^3+3Ep^2-M^2E)\}+M^2E(E+\epsilon)\\
&+E\epsilon(E\epsilon-p^2)-M^2(2E\epsilon-p^2)+2p^2E\epsilon+2M^2E\epsilon\\
&+4p^2E^2-2M^2\epsilon^2+2p^4\big]\cos\theta_{if}\\
F_3=p^2\big[&(\epsilon-E)^2(E^2+3p^2)+(\epsilon-E)\{E^2\epsilon+7Ep^2+3EM^2\}\\
&+M^2\epsilon(E+\epsilon)+p^2(E^2+p^2)+2p^2E^2+2p^2E\epsilon+2p^4\big]\cos\theta_{if'}\\
F_4=p^2\big[&\{3M^2E\epsilon+3E^2(\epsilon-E)^2+4E^2p^2+2p^2E\epsilon+p^2(\epsilon-E)^2+2p^4\}\\
&+(\epsilon-E)\{E^2\epsilon+M^2\epsilon+E(\epsilon-E)^2+2E^3\\
&+\epsilon p^2+6Ep^2\}\big]\cos\theta_{ff'}
\end{aligned}$$

and $X$ is the same as in eqn (17),

$\theta_{if}$, $\theta_{if'}$ and $\theta_{ff'}$ are the angles through which the meson has been scattered.

If we denote by $dQ_0$ the differential scattering cross section in which the radiation damping has been neglected and by $dQ$, the cross section including radiation damping, then from (5)

$$dQ=dQ_0\bigg/\left(1+\frac{b^2}{a^2}\right) \qquad \ldots \quad (19)$$

where $b$ and $a$ are given by (6a) and (6b)

Now $$dQ_0=\frac{\epsilon^2}{(2\pi)^2}\,|H_{if}|^2d\Omega$$

Hence from (16)

$$dQ_0=\frac{g^4}{2E^2}\left\{\frac{A+B\cos\theta}{(X-2p^2\cos\theta)^2}\right\}d\Omega \qquad \ldots \quad (20)$$

where $A$, $B$ and $X$ are given by (17).

Integrating over the entire range of angles, we have from (17) and (18), for the expressions of $a$ and $b$, the following results.

$$a=\pi^3\rho_f g^4\frac{1}{E^2\epsilon^2V^2x^2}\cdot F_a(x) \qquad \ldots \quad (20)$$

where
$$F_a(x) = \{2[(k^2+x^2)\{(1+x^2)+(k^2+x^2)-2(1+x^2)^{1/2}(k^2+x^2)^{1/2}\}$$
$$+k^2\{(1+x^2)+(k^2+x^2)-2(1+x^2)^{1/2}(k^2+x^2)^{1/2}\}+2x^4$$
$$+k^4+2x^2(k^2+x^2)-4(k^2+x^2)^2+4(k^2+x^2)^{3/2}(1+x^2)^{1/2}$$
$$+2\alpha[x^2\{3x^2+(k^2+x^2)^{1/2}(1+x^2)^{1/2}+1\}]\}/(\alpha^2-1)$$
$$-\left(\log\frac{\alpha+1}{\alpha-1}\right)[x^2\{3x^2+(k^2+x^2)^{1/2}(1+x^2)^{1/2}+1\}]$$

where $x$ stands for $p/\mu$; $k=M/\mu$ and $\alpha=\frac{(E-\epsilon)^2}{2p^2}-\frac{M^2}{2p^2}-1$

and
$$b=-\pi^6\rho_f^2\,\frac{g^4}{V^3E^3\epsilon^3}\,\frac{1}{x^4}\cdot F_b(x)$$

where
$$F_b(x)=\left(\log\frac{\alpha+1}{\alpha-1}\right)^2[3k^2(k^2+x^2)^{1/2}(1+x^2)^{1/2}+3(k^2+x^2)$$
$$\{(1+x^2)+(k^2+x^2)-2(1+x^2)^{1/2}(k^2+x^2)^{1/2}\}$$
$$+4x^2(k^2+x^2)+2x^2(k^2+x^2)^{1/2}(1+x^2)^{1/2}+x^2\{(1+x^2)$$
$$+(k^2+x^2)-2(1+x^2)^{1/2}(k^2+x^2)^{1/2}\}+2x^4+\{(1+x^2)^{1/2}$$
$$-(k^2+x^2)^{1/2}\}\{(1+x^2)^{1/2}(k^2+x^2)^{1/2}+k^2(1+x^2)^{1/2}+(k^2+x^2)$$
$$[(k^2+x^2)+(1+x^2)-2(1+x^2)^{1/2}(k^2+x^2)^{1/2}]+2(k^2+x^2)^{3/2}$$
$$+x^2(1+x^2)^{1/2}+6x^2(k^2+x^2)^{1/2}\}]$$

where again, $k$, $X$ and $\alpha$ represent the same values as in $a$.
From the values of $b$ and $a$ as calculated above, we have

$$\frac{b^2}{a^2}=\frac{1}{16(2\pi)^{3}}\,g^4\,\frac{p^2}{(E+\epsilon)^2}\left\{\frac{F_b(x)}{F_a(x)}\right\}^2 \tag{23}$$

It follows from (19) that the total cross section

$$Q=\frac{Q_0}{1+b^2/a}$$

where $Q_0$ can be evaluated from (20) and $b^2/a^2$ has been given by (23).
Integrating over the entire range of angles, we have from (20)

$$Q_0=\pi\cdot\frac{g^4}{4\mu^2(k^2+x^2)}F_a(x)$$

Where $\mu$ is the meson mass and $p$, $x$ and $F_a(x)$ are the same as given by (21) and (23).

## COMPARISON WITH EXPERIMENT

The energy dependence of the total cross section including radiation damping has been shown by means of the graph (Figure 1). We have taken the mass of meson to be $276.m_e$, ($me$ = mass of electron) hence the ratio $M/\mu$ takes the value 6.67. In figure 1, the variation of cross sections with energy

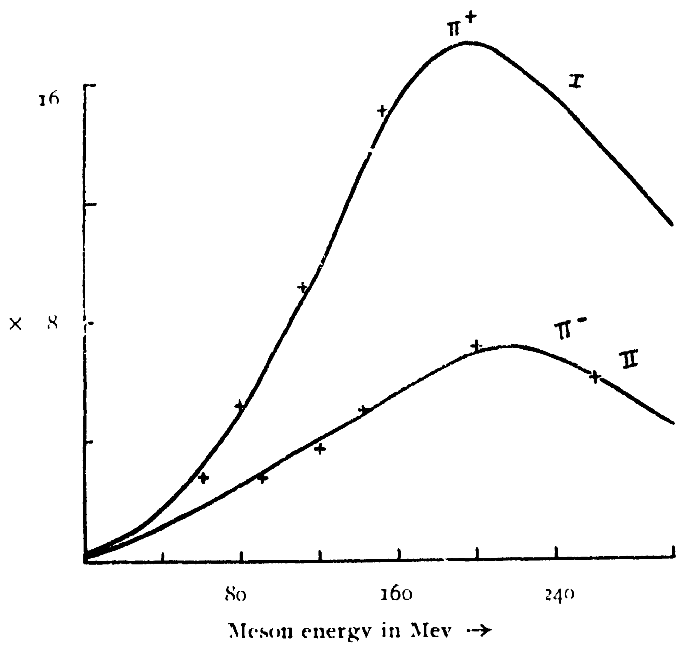


FIG. 1

has been studied in comparison with the experimental results. The general agreement of the theoretical results is fairly good. The effect of radiation damping is predominant at 180 Mev meson energy (kinetic energy). The curve rapidly rises from 2.7 to 15.5 at energies between 60 Mev to 135 Mev energy. The curve falls down from and after 180 Mev showing the non-divergency of the result. The cross points stand for the experimentally observed values. Except at 135 Mev, the curve is seen to be in good agreement with the experiment.

In order to fit the theoretical values in agreement with the experimental ones, a slightly higher value of $g^2$ has been suggested. The value taken is 0.56. This value of $g^2$ is reasonable since it will not disturb the convergency of the higher order cross section terms according to the perturbation theory.

Curve I (in figure 1) represents the total cross section of positive meson scattering by proton. Corresponding experimental values have been shown by cross points.

Curve II (in figure 2) represents the total cross section for the scattering of negative meson by proton. The cross section has been evaluated from the following result of Heueh and Ma (1944).

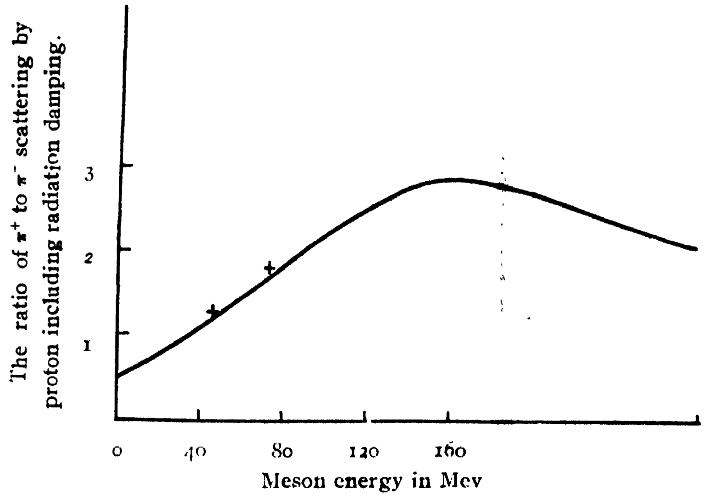


FIG. 2

The total cross section for this case according to the pseudoscalar theory using speudoscalar coupling ($g_2$ coupling) only.

$$\phi = \frac{8\pi}{3p^2} \frac{K_1^2 + K_2^2 + (K_1^2 - K_2^2)^2}{1 + 2(K_1^2 + K_2^2) + K_1^2 - K_2^2}$$

where

$$K_1 = \frac{p(A(E+\epsilon) + BM}{2(E+\epsilon)\{(E+\epsilon)^2 - M^2\}}$$

$$K_2 = \frac{p.(AM + B(E+\epsilon)}{2(E+\epsilon)\{(E+\epsilon)^2 - M^2\}}$$

with

$$A = -g_2^2.E$$

$$B = -g_2^2 M$$

$$C = -p\left[\frac{g_2^2}{m^2}(\epsilon^2 + p^2 + 2E\epsilon)\right]$$

This cross section varies from 2.7 to 6 at energy 80 to 200 Mev with a highest peak of value 7 at 180 Mev energy. The values have been calculated taking $g$ to be 0.56. This gives us an excellent agreement with the experimental result.

In figure 2. the ratio of positive meson scattering to negative meson scattering including radiation damping has been drawn. The values very from 0.5 to 2.3 at energy 0 Mev to 200 Mev. The experimental point 1.8 at 72 Mev is slightly at variance with the theoretical result 1.8 at 80 Mev energy. The experimental point shown by cross point is from the result of Steinberger (1951).

## CONCLUSION

The present paper brings out two points: the influence of radiation reactions begins to assert itself from 200 Mev. The scattering by proton of $\pi^+$ meson is greater than that of $\pi^-$ meson, the difference in the two cases consists in two types of intermediate states, in the former the scattered meson is emitted before the incident meson is absorbed, so there are two mesons in the virtual state, whereas, in the latter case the incident meson is absorbed before the scattered meson is emitted. Because of this difference the matrix element for the former case has larger value than that for the latter. This result is in contradistinction to an observation made by Brueckner (1952) about the weak coupling theory.

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## REFERENCES

Anderson, H. L., 1951, *Phys. Soc. Meeting (Am).*
Basu, D., 1950, *Proc. Roy Ir Acad.*, **53**, 31.
Basu, D, 1951, *Ind. J. Phys.*, **25**, 246.
Bhabha, H. J., 1940, *Proc. Indian. Acad Sci.*, **2**, 247.
Brueckner, A, K., 1952. *Phys. Rev.*, **86**, 106.
Corinalbesi, E. and Field, G., 1949, *Phil. Mag*, **40**. 1150.
Corinaldesi, E. and Field, G., 1950, *Phil. Mag.*, **41**. 364.
Steinberger, Sachs, 1951, *Phys. Rev.*, **82**, 958.
Heitler, W. 1941, *Proc. Camb. Phil. Soc*, **37**, 291.
Heitler, W. and Peng, H. W., 1942, *Proc. Camb. Phil. Soc.*, **38**, 296.
Hsueh and Ma, 1945, *Phys. Rev.*, **67**, 303.
Hsueh and Ma, 1944, *Proc. Comb. Phil. Soc.*, **40**, 167.
Ma S. T, 1943, *Proc. Camb. Phil. Soc.*, **39**, 168.
Wilson, A. H. 1941, *Proc. Camb. Phil. Soc.*, **37**, 1301.